the

# European Set Theory

# Society





1st European Set Theory Conference, Bedlewo, Poland 2007

### "Nobody will expel us from the paradise that Cantor has created" D. Hilbert, 1900

Most people have seen a picture of an older version of the above gentleman. But he was actually quite young when he discovered the fundamental notion of an infinite set and when he proved that the reals are uncountable. Georg Cantor, born in 1845 in St. Petersbourg was only 29 when that proof was published in the Crelle Journal. Cantor was solving a conjecture of Heine on the uniqueness of the presentation of functions as sums of trigonometric series. Thus was created **set theory**, as a way to solve a problem in analysis. Since that time, set theory has lived many lives, some of them parallel and some serial. It has been used for foundations of mathematics and as a way to get closer to the ideas of the Hilbert programme, yet it was finally the Cantor diagonal argument that was in the heart of Gödel's celebrated Incompleteness Theorems [2] which spelled the end of that programme. Set theory has many faces.

### Intradisciplinarity and interdirsciplinarity

*Theorem:* The value of 2<sup>∧</sup>ℵ<sub>0</sub> cannot be calculated in ZFC. (P. Cohen 1963, Fields Medal)



Set theory was taken by Hilbert as a paradigm for the foundations of mathematics and it is in this context that were developed the axioms of set theory. The search for such axioms was long and there have been several interesting candidates. The one considered the main stream today is ZFC, the Zermelo-Fraenkel axioms with the axiom of choice. These axioms can express most modern mathematical objects and therefore it is natural that an investigation of such axioms would have consequences on mathematics in general. It is also natural that research into foundation of mathematics would necessarily lead to logical questions, making set theory part of mathematical logic and giving it connections to computer sciences and philosophy. On the other hand, set theory started as a mathematical investigation of infinite sets and in fact is a subject of mathematics, interested in everything that has to do with infinite sets. From the combinatorics of the infinite to graph theory, topology, measure theory, Banach spaces,

> *Theorem:* If  $2^{\aleph}_{n} < \aleph_{\omega}$  for all n, then  $2^{\aleph}_{\omega} < \aleph_{\omega^{4}}$ (S. Shelah 1984, Israel Prize, Bolyai Prize, Wolf Prize)

C\*-algebras, ergodic theory and group theory, set theorists have over the years not only collaborated and contributed to all these branches of mathematics, but have often been instrumental in developing whole subfields of other fields of mathematics. A recent example is the flourishing subject of nonseparable Banach spaces where major questions have been solved using set theory. Set theory has its own ground as well, large cardinals, inner model theory, forcing axioms and the theory of forcing. If there is a simple phrase to describe set theory, it is the study of the infinite.



#### Set theory and Europe

There were good times, there were bad times. From the beginning, set theory was loved by some (Mittag-Leffler, Hilbert ) and hated by some others (Kronecker). Questions were asked in set theory that literally drove people mad (such as the famous Continuum Hypothesis of Cantor, that stated every infinite subset of the reals is either bijective with **N** or with **R**, hence  $2^{N_0} = \aleph_{T}$ ). Set theory was being developed fast in Germany first of all and then Czechoslovakia, France , Italy, Hungary Poland, Russia, Yugoslavia and elsewhere in



Europe. In some countries this was a strategic development, such as in Poland where the young Polish state between the two wars actively helped build the Polish School of Mathematics, led by a mixture of set theory, topology and measure theory published in the celebrated Fundamenta Mathematicae. As is the case with the rest of the mathematical community, the events leading and during the second world war damaged the set-theoretic community to a terrible extent, with many of our colleagues dying in concentration camps or choosing to end their own life. Escaping the ruins of what has once been, new strong set theoretic communities emerged, in Israel and in the United States. Fraenkel of ZFC moved to Israel (then British Palestine) from Germany and started

#### Abraham Fraenkel

the celebrated Israeli School of Set Theory. John Von Neuman, Stanislaw Ulam and many others moved to the United States and started the equally celebrated USA School. Many set theorists did remain in Europe and continued developing their subject. The postwar years were marked by the unavailability of personal contact or even published work between mathematicians in the Eastern and in the Western block. In set theory we were lucky to have that flying mathematician and enthusiast, Paul Erdös who was the messenger between these two worlds.

## European Set Theory Society

In good times and in bad times, set theory has always existed in Europe and has remained active. Now are the good times. Since the end of the cold war there has been a tremendous increase in the set theoretic activities in Europe, with new centres developing in various countries and old centres flourishing.



#### The INFTY logo



Young Set Theory, Bonn 2011

To give an indication of the image of set theory among young people, a yearly conference called "Young Set Theory" attracts every year about 70 top Ph.D. students. European Science Foundation has funded a large research network called INFTY (2009-2014), led by set theorists in collaboration with other logicians and philosophers. Whole research centres are focused on set theory, such as the Kurt Goedel Research Centre in Vienna.

Nevertheless, we feel that the set theoretic community could become even stronger if we build on the strengths of the community in Europe. Set theory has a very multicultural and diverse history and ambitions and it was in the view of this that we founded the European Set Theory Society. This organisation aims to represent

> all set theorists, not only European, and any set theorist is welcome to join. We have a modest joining fee of 20 euros a year, free for students, unemployed and retired members of the community. Our society has actions and it has dreams. We are called European Set Theory Society because our

actions take place in Europe. Our actions are organisation of conferences, scientific support of the members of the community, and importantly, raising the level of awareness and understanding of research in set theory among other mathematicians. We have just started officially; in 2011 we became a registered charity in the UK. Our financial ability is still quite small, but we intend to build it up and to use that money for prizes, grants and other ways of promoting our subject. Our dreams are very simple, we would like set theory to gain its deserved and historical place as a mainstream subject of mathematics in Europe.

European Set Theory Society has a web site

http://ests.wordpress.com/

#### The Board of Trustees of ESTS consists of:

Prof. Mirna Dzamonja (University of East Anglia, UK) President

Prof. István Juhász (Alfred Renyi Institute of Mathematics and Hungarian Academy of Sciences) Vice-President

Prof. Jouko Väänänen (University of Helsinki, Finland and University of Amsterdam, Netherlands) Treasurer

Prof. Boban Velickovic (University Paris VII, France) Secretary

Prof. Joan Bagaria (University of Barcelona, Spain) Member and Founding President

Prof. Alessandro Andretta (University of Torino, Italy) Member

Prof. Ralf Schindler (University of Muenster, Germany) Member